Paper Reference(s)

6677/01

Edexcel GCE

Mechanics M2

Advanced Level

Monday 13 June 2011 – Morning

Time: 1 hour 30 minutes

Materials required for examination
Mathematical Formulae (Pink)

Items included with question papers
Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

In the boxes on the answer book, write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Mechanics M2), the paper reference (6678), your surname, other name and signature.
Whenever a numerical value of g is required, take $g = 9.8$ m s^{-2}.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
There are 8 questions in this question paper.
The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

1. A car of mass 1000 kg moves with constant speed V m s^{-1} up a straight road inclined at an angle θ to the horizontal, where $\sin\theta = \frac{1}{30}$. The engine of the car is working at a rate of 12 kW. The resistance to motion from non-gravitational forces has magnitude 500 N.

Find the value of V.

(5)

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2. A particle P of mass m is moving in a straight line on a smooth horizontal surface with speed $4u$. The particle P collides directly with a particle Q of mass $3m$ which is at rest on the surface. The coefficient of restitution between P and Q is e. The direction of motion of P is reversed by the collision.

Show that $e > \frac{1}{3}$.

(8)

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3. A ball of mass 0.5 kg is moving with velocity $12\mathbf{i}$ m s^{-1} when it is struck by a bat. The impulse received by the ball is $(-4\mathbf{i} + 7\mathbf{j})$ N s. By modelling the ball as a particle, find

(a) the speed of the ball immediately after the impact,

(4)

(b) the angle, in degrees, between the velocity of the ball immediately after the impact and the vector $\mathbf{i}$,

(2)

(c) the kinetic energy gained by the ball as a result of the impact.

(2)

4.

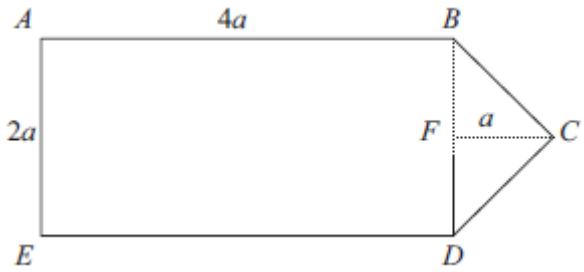

Figure 1

Figure 1 shows a uniform lamina $ABCDE$ such that $ABDE$ is a rectangle, $BC = CD$, $AB = 4a$ and $AE = 2a$. The point F is the midpoint of BD and $FC = a$.

(a) Find, in terms of a, the distance of the centre of mass of the lamina from AE.

(4)

The lamina is freely suspended from A and hangs in equilibrium.

(b) Find the angle between AB and the downward vertical.

(3)

5.

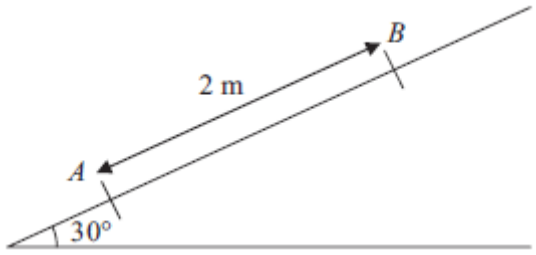

Figure 2

A particle P of mass 0.5 kg is projected from a point A up a line of greatest slope AB of a fixed plane. The plane is inclined at 30° to the horizontal and $AB = 2$ m with B above A, as shown in Figure 2. The particle P passes through B with speed 5 m s^{-1}. The plane is smooth from A to B.

(a) Find the speed of projection.

(4)

The particle P comes to instantaneous rest at the point C on the plane, where C is above B and $BC = 1.5$ m. From B to C the plane is rough and the coefficient of friction between P and the plane is μ.

By using the work-energy principle,

(b) find the value of μ.

(6)

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6. A particle P moves on the x-axis. The acceleration of P at time t seconds is $(t - 4)$ m s^{-2} in the positive x-direction. The velocity of P at time t seconds is v m s^{-1}. When $t = 0$, $v = 6$.

Find

(a) v in terms of t,

(4)

(b) the values of t when P is instantaneously at rest,

(3)

(c) the distance between the two points at which P is instantaneously at rest.

(4)

7.

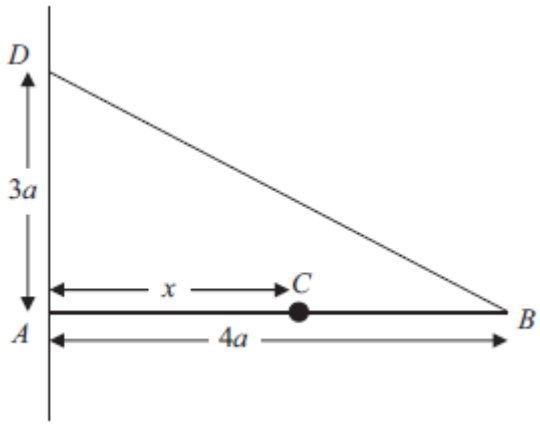


Figure 3

A uniform rod AB, of mass $3m$ and length $4a$, is held in a horizontal position with the end A against a rough vertical wall. One end of a light inextensible string BD is attached to the rod at B and the other end of the string is attached to the wall at the point D vertically above A, where $AD = 3a$. A particle of mass $3m$ is attached to the rod at C, where $AC = x$. The rod is in equilibrium in a vertical plane perpendicular to the wall as shown in Figure 3. The tension in the string is $\frac{25}{4}mg$.

Show that

(a) $x = 3a$,

(5)

(b) the horizontal component of the force exerted by the wall on the rod has magnitude $5mg$.

(3)

The coefficient of friction between the wall and the rod is μ. Given that the rod is about to slip,

(c) find the value of μ.

(5)

8. A particle is projected from a point O with speed u at an angle of elevation α above the horizontal and moves freely under gravity. When the particle has moved a horizontal distance x, its height above O is y.

(a) Show that

$$y = x \tan \alpha - \frac{gx^2}{2u^2 \cos^2 \alpha}.$$

(4)

A girl throws a ball from a point A at the top of a cliff. The point A is 8 m above a horizontal beach. The ball is projected with speed 7 m s^{-1} at an angle of elevation of 45°. By modelling the ball as a particle moving freely under gravity,

(b) find the horizontal distance of the ball from A when the ball is 1 m above the beach.

(5)

A boy is standing on the beach at the point B vertically below A. He starts to run in a straight line with speed v m s^{-1}, leaving B 0.4 seconds after the ball is thrown.

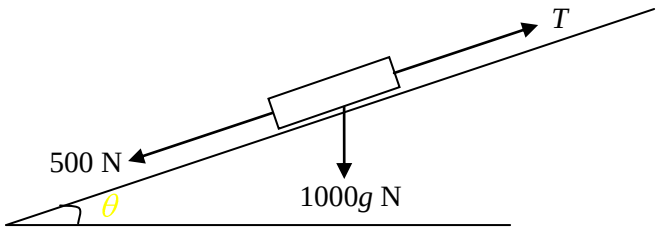
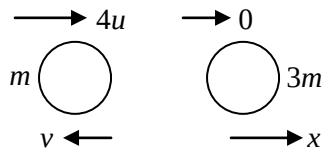
He catches the ball when it is 1 m above the beach.

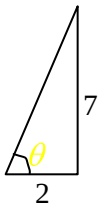
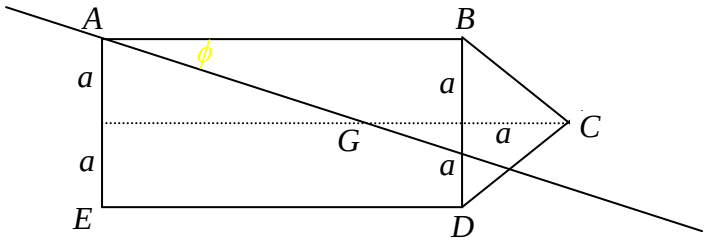
(c) Find the value of v.

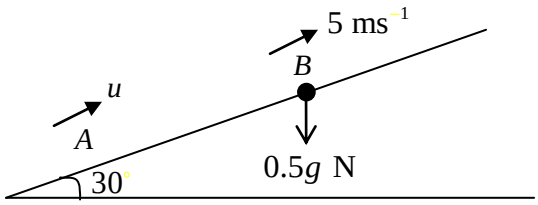
(4)

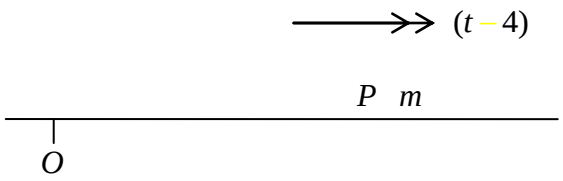
TOTAL FOR PAPER: 75 MARKS

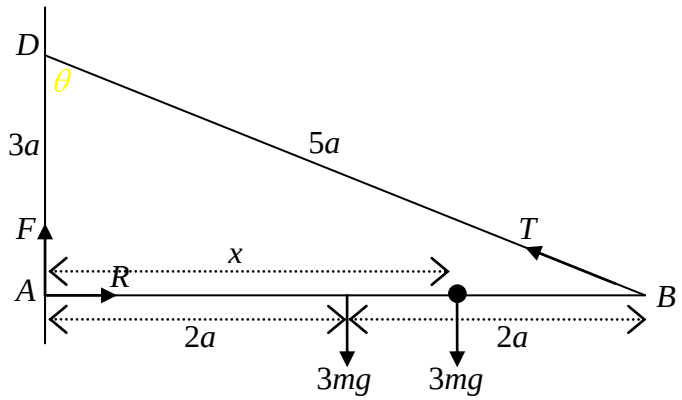
END

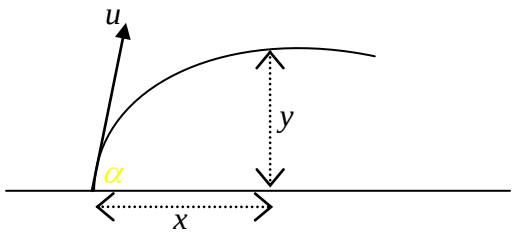
Question Number	Scheme	Marks
1.		
	$12000 = TV$	M1
	$T - 500 - 1000g\sin\theta = 0$	M1 A1
	$V = \dfrac{12000}{500 + 1000 \times 9.8 \times \frac{1}{30}}$	
	$V = 15$ (accept 14.5)	DM1 A1
		(5) **5**
2.		
	$4mu = 3mx - mv$	M1 A1
	$4ue = x + v$	M1 A1
	$4u = 3\ 4ue - v\ - v$	
	$4u = 12ue - 4v$	
	$v = \ 3e - 1\ u$	DM1 A1
	$v > 0 \Rightarrow 3e > 1$	DM1
	$\therefore e > \frac{1}{3}$ **	A1
		(8) **8**

Question Number	Scheme	Marks
3. (a)	$\mathbf{I} = m\mathbf{v} - m\mathbf{u}$ $-4\mathbf{i} + 7\mathbf{j} = 0.5\ \mathbf{v} - 12\mathbf{i}$ $4\mathbf{i} + 14\mathbf{j} = \mathbf{v}$ Speed $= \sqrt{16 + 196} = \sqrt{212}$ m s^{-1} (14.6 or better)	M1 A1 M1 A1 (4)
(b)	 $\tan\theta = \frac{7}{2}$ $\theta = 74.0....$ $\theta = 74°$	M1 A1ft (2)
(c)	Gain in K.E. $= \frac{1}{2} \times 0.5\ \ 212 - 12^2\ \ , \ \ = 17$ J	M1 A1 (2) **8**
4. (a)	 	
	ABDE | *BCD* | Lamina Mass ratio: $8a^2\rho$ | $a^2\rho$ | $9a^2\rho$ 8 | 1 | 9	B1
	Dist of C of M From AE: 2a | $4\frac{1}{3}a$ | $\bar{x}$	B1
	$8 \times 2a + 1 \times \frac{13}{3}a = 9\bar{x}$	M1
	$\bar{x} = \frac{61}{27}a$ (2.26*a*)	A1 (4)
(b)	$\tan\phi = \frac{a}{\frac{61}{27}a} = \frac{27}{61}$	M1 A1 ft
	$\phi = 23.87... = 24°$ (accept 23.9), 0.417 radians	A1 (3) **7**

Question Number	Scheme	Marks
5. (a)		
	$0.5g \times 2\sin 30 = \frac{1}{2} \times 0.5u^2 - \frac{1}{2} \times 0.5 \times 5^2$	M1 A1
	$\frac{1}{4}u^2 = 0.5g + \frac{1}{2} \times 0.5 \times 5^2$	
	$u = 6.7 \text{ m s}^{-1}$ (accept 6.68)	DM1 A1
		(4)
(b)		
	$R = 0.5g\cos 30$	B1
	$F = 0.5g\cos 30 \times \mu$	M1
	Work done by friction $= 1.5F$	
	$\frac{1}{2} \times 0.5 \times 5^2 = 1.5F + 0.5g \times 1.5\sin 30$	M1 A1 A1
	$\mu = \frac{\frac{1}{2} \times 0.5 \times 5^2 - 0.5g \times 1.5\sin 30}{0.5g\cos 30 \times 1.5}$	
	$\mu = 0.40$ (accept 0.4 or 0.405)	A1
		(6)
		10

Question Number	Scheme	Marks
6. (a)	 $\frac{dv}{dt} = t - 4$ $v = \frac{1}{2}t^2 - 4t + c$ $t = 0 \;\; v = 6 \quad \Rightarrow c = 6$ $\therefore v = \frac{1}{2}t^2 - 4t + 6$	 M1 A1 M1 A1 (4)
(b)	$v = 0 \quad 0 = t^2 - 8t + 12$ $t - 6 \;\; t - 2 \;\; = 0$ $t = 6 \;\; t = 2$	M1 DM1 A1 (3)
(c)	$x = \frac{t^3}{6} - 2t^2 + 6t + k$ $x_6 - x_2 = \frac{6^3}{6} - 2\times 6^2 + 6^2 + k$ $-\left(\frac{2^3}{6} - 2\times 2^2 + 6\times 2 + k\right)$ $= -5\frac{1}{3}$ $\therefore$ Distance is $5\frac{1}{3}$m	M1 A1 ft DM1 A1 (4) **11**

Question Number	Scheme	Marks
7. (a)	 M(A) $3mg \times 2a + 3mgx = T\cos\theta \times 4a$ $= \frac{12}{5}aT$ $\frac{12}{5}aT = 6mga + 3mgx$ $T = \frac{25}{4}mg$ $\frac{12}{5}a \times \frac{25}{4}mg = 6mga + 3mgx$ $15a = 6a + 3x$ $x = 3a$ **	M1 A2,1,0 M1 A1 (5)
(b)	R $\rightarrow$ $R = T\sin\theta$ $= \frac{25}{4}mg \times \frac{4}{5}$ $= 5mg$ **	M1 A1 A1 (3)
(c)	R $\uparrow$ $F + \frac{25}{4}mg \times \frac{3}{5} = 3mg + 3mg$ $F = 6mg - \frac{15}{4}mg = \frac{9}{4}mg$ $\mu = \frac{F}{R} = \frac{\frac{9}{4}mg}{5mg} = \frac{9}{20}$	M1 A2,1,0 DM1 A1 (5) **13**

Question Number	Scheme	Marks
8. (a)		
	Horiz: $x = u\cos\alpha t$	B1
	Vert: $y = u\sin\alpha t - \frac{1}{2}gt^2$	M1
	$y = u\sin\alpha \times \frac{x}{u\cos\alpha} - \frac{1}{2}g \times \frac{x^2}{u^2\cos^2\alpha}$	DM1
	$y = x\tan\alpha - \frac{gx^2}{2u^2\cos^2\alpha}$ **	A1
		(4)
(b)	$y = -7$: $-7 = \tan 45x - \frac{gx^2}{2\times 7^2\cos^2 45}$	M1 A1
	$-7 = x - \frac{9.8x^2}{7^2}$	
	$-7 = x - \frac{x^2}{5}$	M1
	$x^2 - 5x - 35 = 0$	
	$x = \frac{5 \pm \sqrt{25 + 4\times 35}}{2}$	M1
	$x = 8.92$ or 8.9	A1
		(5)
(c)	Time to travel 8.922 m horizontally $= \frac{8.922}{7\cos 45} = 1.802...$s	M1
	$v = \frac{8.922}{1.402}$	M1 A1 ft
	$= 6.36$ or 6.4 m s^{-1}	A1
		(4)
		13